CM-18 Lessons in Classical Mechanics

I.1 Review of Newtonian Mechanics

A. K. Kapoor
http://0space.org/users/kapoor
akkapoor@cmi.ac.in; akkhcu@gmail.com

1 A Review of Newtonian Mechanics

Newtonian mechanics will prerequisite for this course. So let us talk about Newtonian mechanics. You have been learning this area of Physics from School days. Please recall the topics you have studied in +2 class and here in different mechanics courses. Each one of you must tell me a topic and some important concepts,laws, or results that you learned in the topic.

$\boxed{\text{Some Text to Be Added}}$

2 Limitations of Newtonian Mechanics

1. One must always write EOM in Cartesian form and then change variables if necessary to a new coordinates such as (r, θ, ϕ).

   For example must write

   $$m\ddot{x} = F_x, \quad m\ddot{y} = F_y, \quad m\ddot{z} = F_z,$$

   where F_x, F_y, F_z are the components of the force. It would be wrong to write.

   $$m\ddot{r} = F_r, \qquad m\ddot{\theta} = F_\theta, \text{etc.}$$

   where F_r, F_θ are radial or r, θ components of the force.

2. In addition to the applied forces or the external forces, one must also include forces of reaction or forces of constraints while setting up the EOM. The forces of constraints become known only after the full solution has been obtained. For example for the pendulum we have take into account of the tension of the string which can be computed only after EOM are solved.

3. For many systems the coordinates and velocities must satisfy constraint relations. For example for a particle moving on the surface of a sphere the constraint relation

   $$x^2 + y^2 + z^2 = R^2$$

   must be imposed separately on the solutions of EOM.

4. Several different types of constraints are possible

   (a) $z = f(x, y)$ particle moves on a surface

   (b) $f(x, y, z, \dot{x}, \dot{y}, \dot{z}) = 0$

   (c) For gas molecules in a cubical container the position coordinates satisfy

   $$-L \leq x \leq L, \qquad -L \leq y \leq L \qquad -L \leq z \leq L$$

   (d) Constraint relations involving only coordinates and possibly time, are called holonomic. These are given by expressions of the form

   $$f_j(\vec{x}_1, \vec{x}_2, \ldots \vec{x}_N, t) = 0, \qquad\qquad j = 1 \ldots m$$

In our course we shall be concerned only with systems having holonomic constraints. For a system with N particles, $3N$ coordinates are needed. If these coordinates satisfy m relations of type $f(\vec{x}, t) = 0$. Only $3N - m$ coordinates will be independent and we say that the system has $3N - m$ degrees of freedom.

The Lagrangian formulation of mechanics allows us to remove redundant coordinates and to deal with independent variables only. It also gives freedom to choose 'any' set of coordinates and use of Cartesian coordinates is not mandatory.

We introduce concept of generalized coordinates by,

(a) $q_1, q_2 \ldots q_n$ are functions of $\vec{x_1}, \vec{x_2} \ldots$

(b) The number q_k is equal to the number of **degrees of freedom**.

(c) All $q's$ are independent

(d) All $\vec{x_\alpha}$ are expressible in terms of $q_1, q_2, \ldots q_k \ldots$

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