

Study Material Part-A
From Newton EOM to Euler Lagrange EOM

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1 A Review of Newtonian Mechanics

Newtonian mechanics will prerequisite for this course. So let us talk about Newtonian mechanics. You have been learning this area of Physics from School days. Please recall the topics you have studied in +2 class and here in different mechanics courses. Each one of you must tell me a topic and some important concepts,laws, or results that you learned in the topic.

Some Text to Be Added

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Contents

§1 Prerequisites 1

§2 Constraints, Degrees of freedom, Generalized coordinates 2

§3 From Newton’s EOM to Euler Lagrange EOM 2

 §3.1 Start with Newton’s Second Law 2

 §3.2 Net force on a particle is a sum of external forces and forces due to constraint 3

 §3.3 Eliminating forces of constraints 3

 §3.4 Express ‘total work done’ in terms of generalised coordinates . . . 3

 §3.5 Bring in kinetic energy 4

 §3.6 Define Generalized force 5

 §3.7 The variations in generalized coordinates are independent 5

§4 Special Cases 5

 §4.1 Case I: Conservative forces 5

 §4.2 Case II: Potential Dependent on Generalised Velocities 6

§5 Points to Remember 7

§6 Learn More 7

§1 Prerequisites

1. We will start with Newton’s laws of motion;
- In general for a physical system there may be *constraints* on the position vectors \vec{r}_α of the particles making up the system. Our discussion will be concerned exclusively with holonomic constraints. These are given by equations of the form

$$\phi_k(\vec{r}_1, \dots, \vec{r}_N, t) = 0, k = 1, \dots$$

I.3 Conservation of Energy

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§1 Lets us discuss

1. Give an example of application of energy conservation.
2. What do you understand by conservation law? When we say something $X(q, p)$ is a constant of motion what exactly it means mathematically?
3. Give examples of a system for which energy is not conserved.
4. Consider an example of a particle moving in one dimension in a potential $V(x)$. Taking total time derivative of total energy, $E = \frac{1}{2}m\dot{x}^2 + V(x)$ we would get

$$\frac{dE}{dt} = m\dot{x}\ddot{x} + \frac{dV(x)}{dx} \frac{dx}{dt}. \quad (1)$$

The right hand side does not seem to become zero!

What is missing?

How do we see that the right hand side of (1) is zero?

§2 Energy conservation

If the Lagrangian does not depend on time explicitly, there is a conservation law and the corresponding conserved quantity will be called as Hamiltonian. The Hamiltonian which coincides with energy ($= KE + PE$) for a mechanical systems. For other system, qualifies to be identified with energy.

If Lagrangian does not contain t explicitly

$$\frac{\partial L}{\partial t} = 0 \quad (2)$$

Cyclic Coordinates, Integration of EOM by Quadratures

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Contents

1	What is a cyclic coordinate?	1
2	Example for Using Conservation laws	2
2.1	Find all cyclic coordinates and conservation laws	2
2.2	Solve for generalized velocities	2
2.3	Integrate the velocities	3
3	Eliminating Cyclic Degrees of Freedom	3
3.1	Routhian	3
3.2	Using Routhian	4
4	Points to Remember	4

1 What is a cyclic coordinate?

A generalised coordinate q is called cyclic if the Lagrangian L is independent of q . In such a case the equation of motion for q

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0. \quad (1)$$

Since q is cyclic, we have

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) = 0 \quad (2)$$

In other words the momentum canonically conjugate to q is a constant of motion.

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Pages from Book by Calkin

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**These pages from Chapter II of the book
by Calkin cover the prerequisites for
PART-A**