Phy 523 PARTICLE PHYSICS
SOLUTIONS PROBLEM SHEET IV

16.16. Relate the matrix elements of the decay $\Delta^{++}, \Delta^{+}, \Delta^{0}$ and Δ^{-} to pions and nucleons assuming isospin invasriance. ($I(\Delta) = 3/2$) .

Solutions:

$\Delta^{++}(I = 3/2; I_z = 3/2)$ can decay to $p + \pi^{+}$ and let the decay rate by given by Γ. I, I_z of Δ^{+}, Δ^{0} and Δ^{-} are $1/2, -1/2$ and $-3/2$ respectively.The final state of the nucleon and the pion is also $I = 3/2$ we have

$$|p\pi^{+} >= |I = 3/2, I_z 3/2 >;$$

$$|p\pi^{0} >= \left(\frac{2}{3}\right)^{1/2} |I = 3/2, I_z = 1/2 > + \left(\frac{1}{3}\right)^{1/2} |I = 1/2, I_z = 1/2 >$$

$$|p\pi^{-} >= \left(\frac{1}{3}\right)^{1/2} |I = 3/2, I_z = -1/2 > - \left(\frac{2}{3}\right)^{1/2} |I = 1/2.I_z - 1/2 >$$

$$|n\pi^{+} >= \left(\frac{1}{3}\right)^{1/2} |I = 3/2, I_z = 1/2 > + \left(\frac{2}{3}\right)^{1/2} |I = 1/2, I_z = 1/2 >$$

$$|n\pi^{0} >= \left(\frac{2}{3}\right)^{1/2} |I = 3/2, I_z = -1/2 > + \left(\frac{1}{3}\right)^{1/2} |I = 1/2, I_z = -1/2 >$$

$$|n\pi^{-} >= |I = 3/2, I_z = -3/2 >$$

Since the final state is only $I = 3/2$, we get for the matrix elements the following

$$< p\pi^{+}|M|\Delta^{++} >=< I = 3/2, I_z = 3/2|M|\Delta^{++} >$$

$$< p\pi^{0}|M|\Delta^{+} >= \left(\frac{2}{3}\right)^{1/2} < I = 3/2, I_z = 1/2|M|\Delta^{+} >$$

$$< n\pi^{+}|M|\Delta^{+} >= \left(\frac{1}{3}\right)^{1/2} < I = 3/2, I_z = 1/2|M|\Delta^{+} >$$

$$< p\pi^{-}|M|\Delta^{0} >= \left(\frac{1}{3}\right)^{1/2} < I = 3/2, I_z = -1/2|M|\Delta^{0} >$$

$$< n\pi^{0}|M|Delta^{0} >= \left(\frac{2}{3}\right)^{1/2} < I = 3/2, I_z = -1/2|M|\Delta^{0} >$$

$$< n\pi^-|M|\Delta^- >=< I = 3/2, I_z = -3/2|M|\Delta^- >$$

Thus the decay rates are

$$\Gamma(\Delta^{++} \rightarrow p\pi^+) = \Gamma$$

(by definition)

$$\Gamma(\Delta^+ \rightarrow p\pi^0) = \frac{2}{3}\Gamma$$

$$\Gamma(\Delta^+ \rightarrow n\pi^+) = \frac{1}{3}\Gamma$$

$$\Gamma(\Delta^0 \rightarrow p\pi^-) = \frac{1}{3}\Gamma$$

$$\Gamma(\Delta^0 \rightarrow n\pi^0) = \frac{2}{3}\Gamma$$

$$\Gamma(\Delta^- \rightarrow n\pi^-) = \Gamma$$

17. Genralised statistics states that isospin wave funtion $\times$ spin wave function $\times$ spatial wave function should be symmetric (antisymmetric) for bosons (fermions) under the interchange of particles. Find the combinations allowed (isospin,spin and spatial) for (a) two pions (b) two nucleons. The spatial wave function can be considered as the orbital angular momentum

Solutions:

Two pions: Three possible isospin states $I = 0, 1, 2$ out of which $I = 1$ is antisymmetric and $I = 0, 2$ are symmetric. Since pions are spinless we have for $I = 1$ odd orbital angular momentum states and for $I = 0, 2$ even orbital angular momentum.

Two Nucleons: $I = 0, 1; S = 0, 1$ $I = 0$ and $S = 0$ are antisymmetric in isospin and spin respctively and $I = 1$ and $S = 1$ are symmetric in isospin and spin respectively. The total wave function is odd under the interchange of the two particle. Thus the allowed angular momenta L are as follows:

$L =$ even for $I = 1, S = 0$ or $I = 0, S = 1$ and $L =$ odd for $I = 1, S = 1$ and $I = 0, S = 0$.

18. Isospin (J^P) of ω^0 is I=1 and 1^-.(It's mass is $780 Mev/c^2$) Show that it can not decay into two pions if isospin is conserved. Given that deuteron is in $l = 0, S = 1$ find the isospin of deuteron.

Solution:

$I=0$ for two pions means isospin is symmetric. Statistics implies that, since pions are spinless $L=J$ is even. Since $J(\omega^0)=1$ the decay can not occur if isospin is conserved in the decay.

19. η^0 is an I=0 and $J^P=0^-$ particle .(It's mass $=550Mev/c^2$) Show that it can not decay to two pions if parity is conserved in the decay.

Solution:

The intrinsic parity of pion is negative.Therefore the final state parity of the two pions is $(-1)^L$ Since the inital angular momentum is 0 the final angular momentum (= the orbital angular momentum of the two spinless pions) $L=0$ and thus the parity of the final two pion state would be positive. Since η is a state of $-$ve parity the decay is forbidden if parity is conserved.

20. ω^0 decays to three pions, $\omega^0\rightarrow\pi^0+\pi^++\pi^-$. Let the average energies of π^+,π^- be $<E_+>,<E_->$ respectively. (a) Write down an expression for the average values $<E_+>$ and $<E_->$ in terms of $|M|^2$ and phase space integral over the three momenta of pions. ($|M|^2$ is the square of the approriate matrix element.) (b) Show that if C or CP is a conserved in the decay $<E_+>=<E_->$.

Solution:

The expression for the average values can be obtained by noting that the probability $P(\pi^+(E_+),\pi^-(E_-),\pi^0(E_0))$ of producing a given state with energy $E_\pm$ is given by

$$P(\pi^+(E_+),\pi^-(E_-),\pi^0(E_0))=\int\frac{d^3p_+d^3p_-d^3p_0}{16E_+E_-E_0M_\omega}\delta^4(P_\omega-P_+-P_--P_0)$$

$$\delta(P_+^0-E_+)\delta(P_0^0-E_-)\delta(P_0^0-E_0)|<\pi^+(P_+),\pi^-(P_-),\pi^0(P_0)|M|\omega^0>|^2$$

(a)Thus

$$<E_\pm>=\frac{\int dE_+dE_-dE_-P(\pi^+(E_+),\pi^-(E_-),\pi^0(E_0))E_\pm}{\int dE_+dE_-dE_-P(\pi^+(E_+),\pi^-(E_-),\pi^0(E_0))}$$

(b) Note

$$|<\pi^+(P_+),\pi^-(P_-),\pi^0(P_0)|M|\omega^0>|^2=|<\pi^-(P_+),\pi^+(P_-),\pi^0(P_0)|M|\omega^0>|^2$$

from C-invariance. Thus

$$< E_+ >= \frac{\int dE_+ dE_- dE_- P(\pi^+(E_+), \pi^-(E_-), \pi^0(E_0))E_+}{\int dE_+ dE_- dE_- P(\pi^+(E_+), \pi^-(E_-), \pi^0(E_0))}$$

$$= \frac{\int dE_+ dE_- dE_- P(\pi^-(E_+)\pi^+(E_-)\pi^0(E_0))E_+}{\int dE_+ dE_- dE_- P(\pi^+(E_-)\pi^-(E_+)\pi(E_0))}$$

Interchanging the integration variables E_+ and E_- in the numerator we get

$$< E_+ >= \frac{\int dE_+ dE_- dE_- P(\pi^-(E_-)\pi^+(E_+)\pi^0(E_0))E_-}{\int dE_+ dE_- dE_- P(\pi^+(E_-)\pi^-(E_+)\pi(E_0))}$$

$$=< E_- >$$

which proved the desired result.