There is a symmetry transformation that connects equations of motion under the two most important force laws of Kepler's and Hooke's law. Here is the statement from the reference given at the end.

Bohlin's Theorem Suppose a point in the complex plane moves following Hooke's law

$$\frac{d^2w}{dt^2} = -\frac{k}{w} = -Cw \tag{1}$$

Square w and consider a point following trajectory $z(\tau(t)) = [w(t)]^2$, with $\frac{d\tau}{dt} = |w|^2$ where a new time τ has been chosen in order to grant law of areas. Then $z(\tau)$ will satisfy the gravitational law:

$$\frac{d^2z}{d\tau^2} = -\frac{k}{m}\frac{z}{z^3} = -\tilde{C}\frac{z}{z^3} \tag{2}$$

where $\tilde{C} = 2(|w'(0)|^2 + |w(0)|^2)$.

Reference

Maria Luisa Saggio, *Eur. J. of Physics* **34** (2013) 129-137