VS-02 Problem Set

Linear Independence, Basis and Dimension

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⊘ Write short answers to the following questions.

[1] Prove that the set

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

is a basis in the vector space of 2×2 real matrices with $\mathbb{R}$ as the field of scalars.

[2] . Show that the Pauli matrices, $\sigma_x, \sigma_y, \sigma_z$, do not form a basis for the vector space of all 2×2 complex matrices with $\mathbb{C}$ as the field of complex numbers.

[3] Show that the vectors $(1, 1, 1, -1), (1, 1, -1, 1), (1, -1, 1, 1), (4, 2, 2, 0)$ cannot be a basis in $\mathbb{R}^4$ because the vectors are not linearly independent.

[4] The set of vectors $\{(1, 1, 1, 1), (1, -1, 1, 1), (1, 1 - 1, 1)\}$ is linearly independent in Rbb^4 but is not a basis. WHY ?

[5] Prove that a set of three vectors $\{\vec{u}, \vec{v}, \vec{w}\}$ forms a basis if and only if $\vec{u} \cdot \vec{v} \times \vec{w}$ is nonzero.

[6] Generalize the result of $\mathcal{Q}[5]$ above to $\mathbb{R}^n$ and give a short proof.

[7] Let $\mathcal{V} = \mathbb{R}$ and $\mathcal{F} = \mathbb{R}$. Is the set $\{1, \sqrt{2}\}$ a linearly independent set?

[8] Let $\mathcal{V} = \mathbb{R}$ and $\mathcal{F} = \mathbb{Q}$ Is the set $1, \sqrt{2}$ a linearly independent set? WHY?

[9] Is the set $\{1 + i, 1 - i\}$ a linearly independent set in the vector space $\mathbb{C}$? The field of scalars is given to be the set $\mathbb{R}$ of real numbers.

[10] Give a basis for vector space $\mathbb{C}$ on field $\mathbb{C}$ of scalars. Also give a basis for vector space $\mathbb{C}$ with real numbers $\mathbb{R}$ as the field of scalars.