VS-06 Problem Set

Matrix Representation for Vectors and Operators

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[1] Let an arbitrary vector in $\mathbb{C}^5$ be written as $f = (\xi_1, \xi_2, ..., \xi_5)$. Give the matrix representation for the following operatotrs R, S, T, I, U dfined below.

(a) $Rf = (\xi_5, \xi_4, \xi_3, \xi_2, \xi_1)$

(b) $Sf = (\xi_1 + \xi_2, \xi_1 - \xi_2, \xi_3 + \xi_4, \xi_3 - \xi_4, \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5)$

(c) $Tf = (0, \xi_1, \xi_2, \xi_3, \xi_4)$

(d) $If = f$, I = identity operator.

(e) $Uf = 0$, U = zero operator.

[2] An operator T is defined below on the vector space of $\mathscr{L}^2[-\pi, \pi]$.

$$Tf(x) = f(x + \theta)$$

Find the matrix representing the operator T with respect to the basis $\{\phi_n | n = 0, 1, 2, ...\}$, where

$$\phi_0 = 1; \phi_1 = \sin x; \phi_2 = \cos x; \phi_3 = \sin 2x; \phi_4 = \cos 2x;$$

and in general,

$$\phi_{2m-1} = \sin(mx), \phi_{2m} = \cos(mx).$$

[3] Let $\mathscr{P}_n$ be the vector space of all polynomials of degree n. Define operators A and D by

$$Ap(x) = p(x + 1); \qquad Dp(x) = \frac{\partial p(x)}{\partial x}$$

Selecting $1, x, x, x, ..., x, ..$ as a basis

(a) Find the matrix representing the operators A and D.

(b) using the above basis prove that

$$A = 1 + D/1! + D^2/2! + ... + D^n/n!$$

[4] Let $\mathscr{P}$ be the vector space of all polynomials of arbitrary degree. Define operators D and X by

$$Dp(x) = \frac{dp(x)}{dx}; \qquad Xp(x) = xp(x)$$

Using a basis or otherwise show that $XD - DX = -I$, where I is identity operator.

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