Frobenius Method of Series Solution

The Point at Infinity

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Abstract

A method to obtain series solution of a linear differential equation in negative powers of x is described.

For a second order linear differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = 0 \tag{1}$$

sometimes instead of a series solution in powers of x, it may be useful to expand in negative powers of x:

$$y(x, c) = x^c \sum_{n=0}^{\infty} a_n x^{-n} \tag{2}$$

This results on convergence etc. of this type of solutions are conveniently obtained by changing the independent variable from x to $t = 1/x$. The differential equation Eq.(1) written in terms of t becomes

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \tilde{p}(t)\frac{\mathrm{d}y}{\mathrm{d}t} + \tilde{q}(t)y = 0 \tag{3}$$

where

$$\tilde{p}(t) = \frac{2}{t} - \frac{1}{t^2}p(t); \tilde{q}(t) = \frac{1}{t^4}q(1/t) \tag{4}$$

The behaviour of the series solution at $t = 0$ gives the answer for the behaviour of the solution in the inverse powers of x.