SUNDAY PHYSICS --- Quantum Mechanics

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Multiple Choice Questions

January 8, 2023

Sunday Physics — QM (MCQ)

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$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(x_1, x_2) \longrightarrow \quad x = x_1 - x_2$$

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$H = \frac{p^2}{2\mu} + \frac{P^2}{2M} + V(x, X)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$M = m_1 + m_2$$

Consider two coupled harmonic oscillators of mass m each. The Hamiltonian describing the oscillators is

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}m\omega^2(\hat{x}_1^2 + \hat{x}_2^2 + (\hat{x}_1 - \hat{x}_2)^2).$$

The eigenvalues of $\hat{H}$ are given by (with n_1 and n_2 being non-negative integers)

(a) $E_{n_1,n_2} = \hbar\omega(n_1 + n_2 + 1)$

(b) $E_{n_1,n_2} = \hbar\omega(n_1 + \frac{1}{2}) + \frac{1}{\sqrt{3}}\hbar\omega(n_2 + \frac{1}{2})$

(c) $E_{n_1,n_2} = \hbar\omega(n_1 + \frac{1}{2}) + \sqrt{3}\hbar\omega(n_2 + \frac{1}{2})$ ✓

(d) $E_{n_1,n_2} = \frac{1}{\sqrt{3}}\hbar\omega(n_1 + n_2 + 1)$

Option c

Q.No:12 JEST-2019

$$V(x_1, x_2) = \frac{1}{2} m\omega^2 \left(x_1^2 + x_2^2 + (x_1 - x_2)^2 \right)$$

$$= \frac{1}{2} m\omega^2 \left(\frac{1}{2}(x_1 + x_2)^2 + \frac{1}{2}(x_1 - x_2)^2 + (x_1 - x_2)^2 \right)$$

$$= \frac{1}{2} m\omega^2 \left(\frac{1}{2}(x_1 + x_2)^2 + \frac{3}{2}(x_1 - x_2)^2 \right)$$

$X = (x_1 + x_2)/2$, $x = (x_1 - x_2)$ $\quad M = 2m$, $\mu = m/2$

$$H = \frac{p^2}{2\mu} + \frac{P^2}{2M} + \frac{1}{4} m\omega^2 (4X^2) + \frac{3}{4} m\omega^2 (x_1 - x_2)^2$$

$$= \frac{p^2}{2\mu} + \frac{3}{2} \mu\omega^2 x^2 + \frac{P^2}{2M} + \frac{1}{2} M\omega^2 X^2$$

$$= \frac{p^2}{2\mu} + \frac{1}{2} \mu \Omega^2 x^2$$

$$\Omega = \sqrt{3}\,\omega$$

$$\hbar (n_1 + 1/2) \sqrt{3}\,\omega + \hbar (n_2 + 1/2)\,\omega$$

option (c) is the correct answer

Consider a quantum particle of mass m and charge e moving in a two dimensional potential given as:

$$V(x,y) = \frac{k}{2}(x-y)^2 + k(x+y)^2.$$

The particle is also subject to an external electric field $\vec{E} = \lambda(\hat{i} - \hat{j})$, where λ is a constant. $\hat{i}$ and $\hat{j}$ corresponds to unit vectors along x and y directions, respectively. Let E_1 and E_0 be the energies of the first excited state and ground state, respectively. What is the value of $E_1 - E_0$?

λ → will not contribute because in electric field all energy levels are shifted by same amount

(a) $\hbar\sqrt{2k/m}$

(b) $\hbar\sqrt{2k/m} + e\lambda^2$

(c) $3\hbar\sqrt{2k/m}$

(d) $3\hbar\sqrt{2k/m} + e\lambda^2$

Option a

Q.No:1 TIFR-2014

A particle of mass m and charge e is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x) = \frac{1}{2}kx^2 - eEx$$

If the electric field is suddenly switched off, then the particle will

(a) make a transition to any harmonic oscillator state with $x = -eE/k$ as origin without emitting any photon.

(b) make a transition to any harmonic oscillator state with $x = 0$ as origin and absorb a photon.

(c) settle into the harmonic oscillator ground state with $x = 0$ as origin after absorbing a photon.

(d) oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

Option b

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{k}{2}(x-y)^2 + k(x+y)^2$$

$X = (x+y)/2 \quad q = (x-y) \quad M = 2m \quad \mu = \frac{m}{2}$

$$H_0 = \frac{P^2}{2M} + \frac{p^2}{2\mu} + \frac{k}{2}q^2 + k4X^2 \qquad \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$= \frac{P^2}{2M} + \frac{1}{2}M\Omega^2 X^2 \qquad 4k = \frac{1}{2}M\Omega^2 = m\Omega^2$$

$$+ \frac{p^2}{2\mu} + \frac{1}{2}\mu\,\omega^2 q^2 \qquad \Omega = 2\sqrt{k/m}$$

$\vec{E}, \quad V = -\vec{E}\cdot\vec{r} = -\lambda(x-y)$

$\frac{1}{2}\mu\omega^2 = k/2 \Rightarrow \omega^2 = k/\mu = (2k/m)$

$E = E_1 + E_2$

$E_X = (n_1 + \frac{1}{2})\hbar\, 2\sqrt{k/m}$

$E_q = (n_2 + \frac{1}{2})\hbar\sqrt{(2k/m)}$

$(n_1, n_2) = (1,0)$ ✗ ; $= (0,1)$ ✓ ; $(1,1)$ ✗

$E_0 = \frac{1}{2}\hbar(= + =)$

$E_1 = \frac{1}{2}\hbar(= + =) + \hbar\sqrt{(2k/m)}$

$$E_1 - E_0 = \hbar\sqrt{(2k/m)}$$

OPTION

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x$$

$$= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2\left(x + \frac{\lambda}{\sqrt{m}\,\omega}\right)^2 - \frac{1}{2}\frac{m\omega^2\lambda^2}{m\omega^2}$$

C

$x = X$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dX^2} + \frac{1}{2}m\omega^2 X^2 + C$$

$$E_n = (n + \tfrac{1}{2})\hbar\omega + C$$

Q.No:2 TIFR-2016

Consider two spin-$1/2$ identical particles A and B, separated by a distance r, interacting through a potential

Pauli Principle

$$V(r) = \frac{V_0}{r}\vec{S}_A.\vec{S}_B$$

where V_0 is a positive constant and the spins are $\vec{S}_{A,B} = \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of the Pauli spin matrices. The expectation values of this potential in the spin-singlet and triplet states are

(a) Singlet: $\frac{V_0}{3r}$, Triplet: $\frac{V_0}{r}$

(b) Singlet: $-\frac{3V_0}{r}$, Triplet: $\frac{V_0}{r}$

(c) Singlet: $\frac{3V_0}{r}$, Triplet: $-\frac{V_0}{r}$

(d) Singlet: $-\frac{V_0}{r}$, Triplet: $\frac{3V_0}{r}$

Option b

$$V = \frac{V_0}{\hbar^2} \vec{S}_A \cdot \vec{S}_B$$

$$\tfrac{1}{2} \oplus \tfrac{1}{2} = 1, 0$$

$S \to 1$ $\quad \hat{S}^2 \to 1(1+1) = 2\hbar^2$ triplet

$S = 0$ $\quad \hat{S}^2 \to 0$ singlet

$$\bar{S}_A + \bar{S}_B = \bar{S}$$

$$\vec{S}^2 = S_A^2 + S_B^2 + 2\vec{S}_A \cdot \vec{S}_B$$

$$\vec{S}_A \cdot \vec{S}_B = \tfrac{1}{2}(\vec{S}^2 - S_A^2 - S_B^2)$$

Triplet $\to \tfrac{1}{2}(2 - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1))$

$$= \tfrac{1}{2}(2 - 3/2) = 1/4$$

Singlet $\quad \bar{S}_A \cdot \bar{S}_B \to \tfrac{1}{2}(0 - 3/4 - 3/4) = -3/4$

$$\bar{\sigma}_A \cdot \bar{\sigma}_B / 4 = \begin{cases} 1/4 & \text{triplet} \\ -3/4 & \text{Singlet} \end{cases}$$

$$\bar{\sigma}_A \cdot \bar{\sigma}_B \to \begin{cases} 1 & \text{triplet} \\ -3 & \text{singlet} \end{cases}$$

Q.No:3 TIFR-2017

A quantum mechanical system which has stationary states $|1\rangle, |2\rangle$ and $|3\rangle$, corresponding to energy levels $0\,\mathrm{eV}, 1\,\mathrm{eV}$ and $2\,\mathrm{eV}$ respectively, is perturbed by a potential of the form

$$\hat{V} = \varepsilon|1\rangle\langle 3| + \varepsilon|3\rangle\langle 1|$$

where, in eV, $0 < \varepsilon \ll 1$. The new ground state, correct to order ε, is approximately.

(a) $\left(1 - \frac{\varepsilon}{2}\right)|1\rangle + \frac{\varepsilon}{2}|3\rangle$

(b) $|1\rangle + \frac{\varepsilon}{2}|2\rangle - \varepsilon|3\rangle$

(c) $|1\rangle + \frac{\varepsilon}{2}|3\rangle$

(d) $|1\rangle - \frac{\varepsilon}{2}|3\rangle$

Option d

Q.No:4 TIFR-2018

$H = H_0 + \lambda H'$ $\lambda \to \varepsilon$

$H_0 u_n = E_n^{(0)} u_n$

$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \dots$ ✓

$H\psi = E_n \psi$

$\psi = \psi^{(0)} + \lambda \psi^{(1)} + \dots$

$(H_0 + \lambda H')(\psi^{0} + \lambda \psi^{(1)}) = (E^{(0)} + \lambda E^{(1)})(\psi^{(0)} + \lambda \psi^{(1)})$

Indep of λ $H_0 \psi^{(0)} = E^{(0)} \psi^{(0)}$

1st power of λ $H' \psi^{(0)} + H_0 \psi^{(1)} = E^{(0)} \psi^{(1)} + E^{(1)} \psi^{(0)}$

$\Longrightarrow (H_0 - E^{(0)}) \psi^{(1)} = (E^{(1)} - H') \psi^{(0)}$

IMP To find $\psi^{(1)} \to$ expand in $\psi_m^{(0)}$, $m = 1, 2, 3, \dots$ (complete set)

To calculate correction to E_n

$$\psi^{(1)} = \sum_{m \neq n} c_m u_m$$

$$(H_0 - E_n^{(0)}) \sum_{m \neq n} c_m u_m = (E_n^{(1)} - H') u_n$$

$$\sum_{m \neq n} c_m (E_m^{(0)} - E_n^{(0)}) u_m = (E_n^{(1)} - \hat{H}') u_n$$

operator

(1) Take scalar product with u_k $k \neq n$

(2) Take scalar product with u_n!! (n)

1 gives (L.H.S ≠ 0) for $m = k \Rightarrow c_k (E_k - E_n^{(0)}) = -\langle k | \hat{H}' | n \rangle$

$$c_k = -\langle k | \hat{H}' | n \rangle / (E_k^{(0)} - E_n^{(0)})$$

(2) Gives LHS = 0 for all $m \Rightarrow E_n^{(1)} - \langle n | H' | n \rangle = 0$

Final formula for first order Correction

$$E_n^{(1)} = \langle n | H' | n \rangle$$

$$E_n \simeq E_n^{(0)} + E_n^{(1)}$$

First order correction to wave function of n^{th} level

$$\psi_n^{(1)} = \sum_{m \neq n} c_m u_m = \sum_{k \neq n} \frac{\langle n | H' | k \rangle}{E_k^{(0)} - E_n^{(0)}} u_k$$

Also second order correction to non degenerate level

$$E_n^{(0)} = \sum_{k \neq n} \frac{|\langle n | H' | k \rangle|^2}{E_k^{(0)} - E_n^{(0)}} u_k$$

A particle of mass m moves in a two-dimensional space (x, y) under the influence of a Hamiltonian

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{4}m\omega^2(5x^2 + 5y^2 + 6xy)$$

Find the ground state energy of this particle in a quantum-mechanical treatment.

Ans

Q.No:5 TIFR-2019

A system of two spin-1/2 particles 1 and 2 has the Hamiltonian

$$\hat{H} = \epsilon_0 \hat{h}_1 \otimes \hat{h}_2$$

where

$$\hat{L}_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \hat{L}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and ϵ_0 is a constant with the dimension of energy. The ground state of this system has energy

(a) $\sqrt{2}\epsilon_0$

(b) 0

(c) $-2\epsilon_0$

(d) $-4\epsilon_0$

Option c

Q.No:6 TIFR-2021

What are the energy eigenvalues for relative motion in one-dimension of a two-body simple quantum harmonic oscillator (each body having mass m) with the following Hamiltonian?

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{m} + \frac{1}{2}m\omega^2(x_1 - x_2)^2$$

(a) $\sqrt{2}\left(n + \frac{1}{2}\right)\hbar\omega$

(b) $\left(n + \frac{1}{2}\right)\hbar\omega$

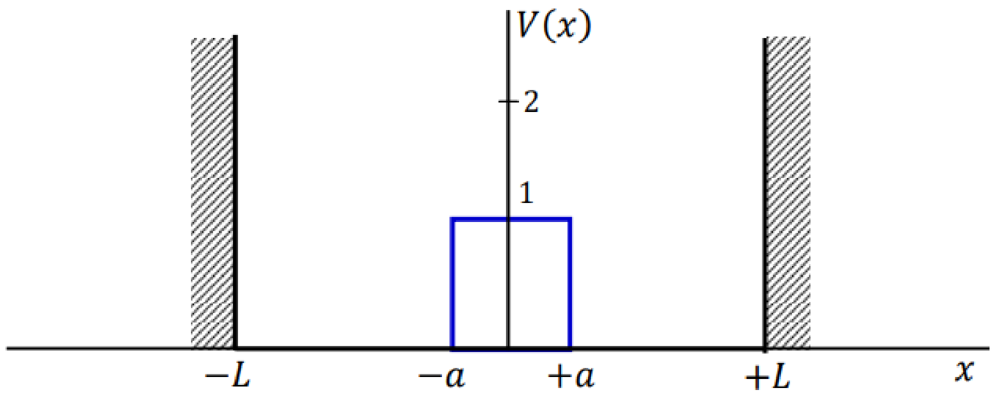
(c) $\frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right)\hbar\omega$

(d) $\sqrt{\frac{3}{2}}\left(n+\frac{1}{2}\right)\hbar\omega$

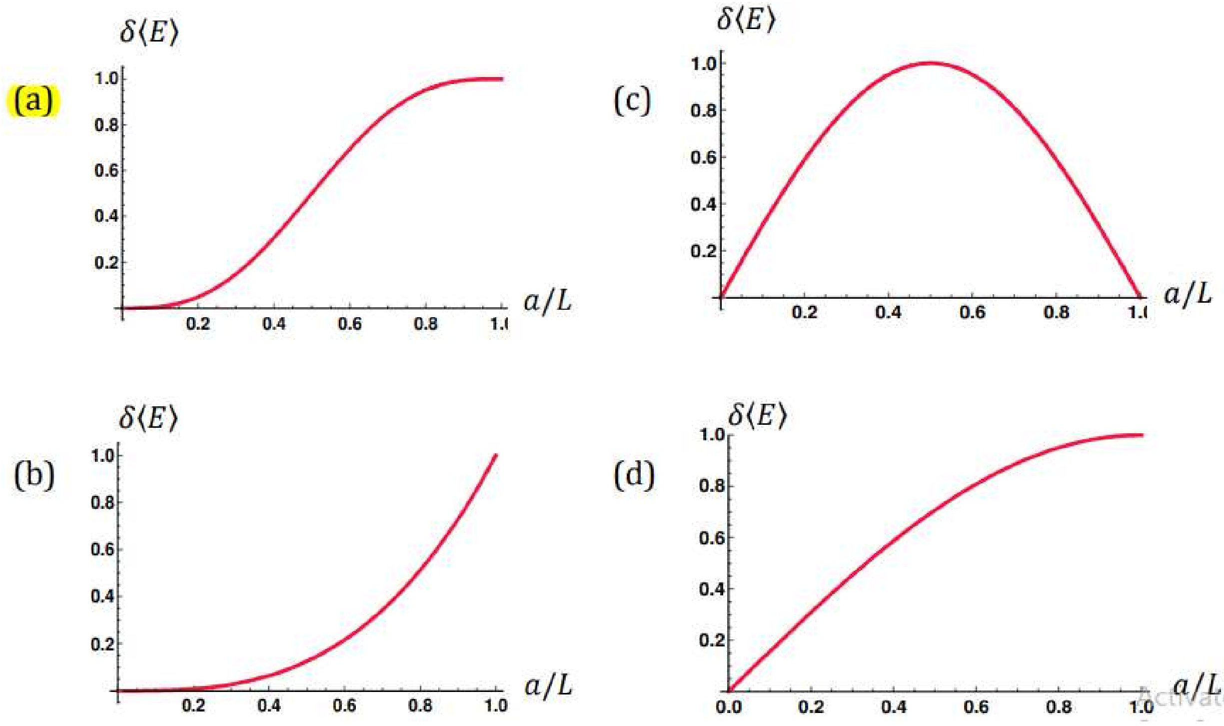
Option a

Q.No:7 TIFR-2021

A particle of mass m, confined in a one-dimensional box between $x=-L$ and $x=L$, is in its first excited quantum state. Now, a rectangular potential barrier of height $V(x)=1$ and extending from $x=-a$ to $x=a$ is suddenly switched on, as shown in the figure below.



Which of the following curves most closely represents the resulting change in average energy $\delta\langle E\rangle$ of the system when plotted as a function of a/L, immediately after the barrier is created?



Option a

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About Us

Our Method